X-512-65-253

FACILITY FORM 602

N66-11202 (ACCESSION NUMBER) (THRU)

36 (PAGES) 1 (CODE)

(NASA CR OR TMX OR AD NUMBER) 16 (CATEGORY)

NASA TM X-55306

EFFECTS OF PROPAGATION TIMES ON THE STEERING OF A NARROW-BEAM LASER TO A DEEP-SPACE VEHICLE

GPO PRICE $

CFSTI PRICE(S) $

Hard copy (HC) 2.00

Microfiche (MF) .50

ff 653 July 65

JUNE 1965

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-512-65-253

EFFECTS OF PROPAGATION TIMES ON THE STEERING OF A NARROW-BEAM LASER TO A DEEP-SPACE VEHICLE

by

K. E. Peltzer
Technical Staff
Manned Flight Support Office

Goddard Space Flight Center
Greenbelt, Maryland

ABSTRACT

11202

This paper deals with one (1) aspect of the problem associated with steering a narrow-beam LASER for communications between a planetary deep-space vehicle and an earth ground station or relay station. The area investigated is that of spacial beam offset.

The introduction is a fundamental discussion of the astronomy of our solar system and is presented in order that an appreciation of the distance from earth to the other planets can be obtained.

Section I presents a derivation of optical refraction using Snell's Law. Expressions are given for the amplitude and phase of light that has undergone refraction through a medium of known thickness and known index of refraction.

Section II gives a classical derivation of the retarded vector potential and is presented as a foundation for the work in Section III. In addition, the radiation vectors as derived by Scknelknoff are defined and the phase term is discussed in light of the problem under discussion.

Section III presents the theory underlying Spacial Beam Offset and an expression is developed that expresses the amount of offset required. For deep ranges, it is shown that the amount of spacial beam offset required does not depend on range, but depends only on the velocity of the spacecraft, aspect angle, and beamwidth of the LASER. This derivation neglects the effect of the atmosphere and considers only the propagation time effects.

The implication of this paper is that under certain geometry, a LASER would not communicate in real time angle with a deep-space vehicle. A narrow-beam LASER would have to be positioned ahead or behind the vehicle at the time of transmission in order to insure that the maximum of the LASER far-field pattern intersects the spacecraft for an optimum communications link.

Author

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

EFFECTS OF PROPAGATION TIMES ON THE STEERING OF A NARROW-BEAM LASER TO A DEEP-SPACE VEHICLE

INTRODUCTION

Man's curiosity insures that the moon will only be a stepping stone in the conquest of knowledge concerning our solar system. A manned mission to the moon is the easiest conquest that can be made outside of the Earth's gravitational field. Most certainly, once this mission is completed, systems for taking us to one of the nearby planets will be off of the drawing boards and in the design stage. In order to understand the problems involved in a manned mission to one of the planets in our solar system, it is necessary to consider the motions of the other planets relative to the sun.

The solar system consists mostly of empty space and what material it does contain belongs primarily to the sun. This material consists of nine (9) planets, their 31 known satellites, thousands of planetoids and comets, and billions of meteorites.

A scale model of the solar system's ten (10) most important bodies (sun and nine (9) planets) where 10,000 miles equals 1 inch, would find the sun about 7.5 feet in diameter and the Earth 3/4 inch in diameter placed 775 feet away. Mercury would be 1/3 inch in diameter, 300 feet away. Jupiter is the largest planet and would be represented by a 9-inch sphere about 3/4 mile from the sun. Pluto is the farthest out and would be about 1/2 the size of the Earth at 5-1/2 miles from the sun in the scale model.

The period of the planets rotation about the sun varies from 88 days for Mercury to 248 years for Pluto. The first person to adequately state the laws of planetary motion was Johannes Kepler. These laws as originally stated are given below:

<u>Law I</u> The orbit of any planet is an ellipse with the sun at one focus.

<u>Law II</u> The line joining sun and planet will sweep over equal areas in equal intervals of time.

<u>Law III</u> The square of the period of revolution of any planet is equal to the cube of its mean distance.

Kepler's laws were originally applied to the relative motion of any planet about the sun. It has been shown that these three (3) laws can be generalized to describe the relative motion of any two (2) mutually revolving bodies.

In order to determine the problems of steering a narrow-beam LASER for communications with a spacecraft on a planetary mission, one must consider the relative motion of the planets to the earth in order to determine the optimum launch time for intersection of the planet and spacecraft. It is difficult to show the entire solar system on a fixed-scale model that can be placed in a report. However, planetary distances and periods can be summarized in table form as shown in Table I.

Table I

Planet	Mean Distance In Miles	Orbital Eccentricity	Period (Years)
Mercury	36×10^6	.2056	.241
Venus	67×10^6	.0068	.615
Earth	92×10^6	.0167	1
Mars	142×10^6	.0934	1.881
Jupiter	483×10^6	.0484	11.862
Saturn	886×10^6	.0557	29.458
Uranus	1782×10^6	.0472	84.013
Neptune	2792×10^6	.0086	164.793
Pluto	3664×10^6	.2502	248.430

Mars is the object of the greatest popular interest because of the question of the possibility of supporting some form of life. Table I indicates that the ratio of periods for Mars and Earth is 1.88. Mars is best seen from Earth at the times of its opposition (when it is nearest to Earth) and oppositions of Mars occur every 780 days. The opposition distance can vary from 34,600,000 miles to 62,900,000 miles due to the eccentricity of its orbit. The most favorable oppositions occur at intervals of 15 to 17 years (as in 1939, 1956, 1971) and always in August or September for the Earth is nearest Mars' perihelion on August 28. The relative position of Mars' aphelion and perihelion is shown in Figure 1.

The problem of launching a manned expedition to Mars from Earth must take into account the relative position of these two (2) planets at the time of launch. It is not the purpose of this paper to optimize the flight trajectory, but a manned mission to Mars would certainly have unique communications and

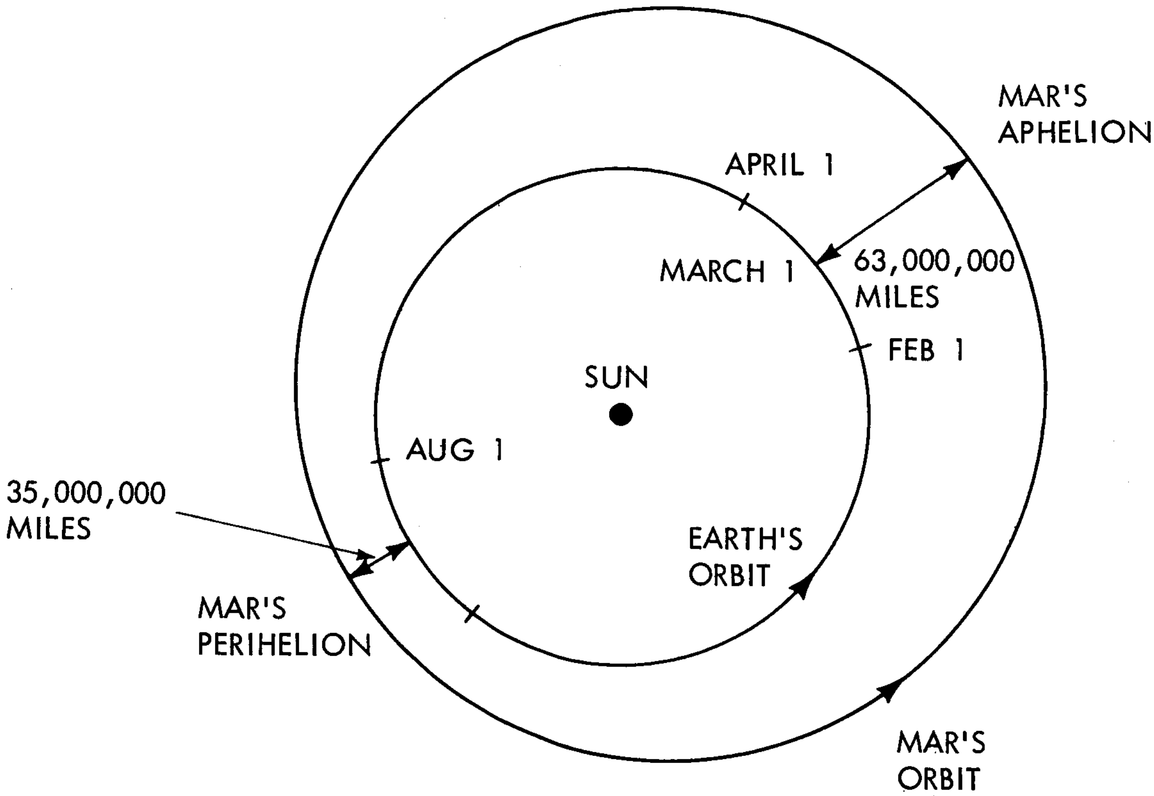


Figure 1–Relative Position of Mar's Aphelion and Perihelion.

tracking requirements. The ranges involved and the manned aspect of the mission place unusual requirements for two-way communications. It seems reasonable to assume that there would be a necessity for transmitting high-bit-rate telemetry from the spacecraft. In addition, continual tracking of the vehicle seems mandatory during periods of engine burn with the capability for high angular resolution. These two (2) requirements seem to point out the LASER as a potential source for communications and tracking, at least until such time as a highly maneuverable spacecraft is developed. Both the up-link and down-link must be considered. It will be shown that precise spacecraft velocity and angle information would be mandatory for positioning a large diffraction-limited telescope or a LASER transmitter to a predetermined position in space.

It is well known that a coherent optical beam will undergo some dispersion through the atmosphere due to the non-linearity of the index of refraction. At this time, exact values of the amount of dispersion are not well known. It is important to consider this since a narrow coherent beam will be enlarged after going through the Earth's atmosphere. The beam will bend away from the original ray path affecting the angular accuracy unless exact information concerning the amount of bending is known. If one considers the atmosphere as a large number of concentric shells, with each shell having a slightly different index of refraction, it can be seen that the amount of beam spreading and bending is a function of the elevation angle of the LASER on earth. A LASER mounted on a deep-space vehicle pointed at Earth, will have an angle of incidence considered closer to perpendicular to the concentric shells for most cases. This is not true for a LASER on the Earth since the angle of incidence with the lower boundaries of the atmosphere is a function of elevation angle of the LASER.

I. REFRACTION PRINCIPLE OF OPTICAL BEAM DEVIATION

The principle of refraction is based on the fact that a beam of light crossing a boundary between two (2) media of different indexes of refraction will be deviated from the incident direction. The amount of deviation is proportional to the indexes of refraction of the two (2) media.

If θ_1, η_1, θ_2, and η_2 are the angles which the beam direction makes with the normal and the index of refraction of the incident and refracted medium, then from Snell's law

$$\theta_2 = \sin^{-1}\left[\frac{\eta_1}{\eta_2}\sin\theta_1\right] \tag{1}$$

Thus, any change in θ_1 will cause a change in θ_2. Also

$$(\eta_2 + \Delta\eta_2)\sin(\theta_2 + \Delta\theta_2) = \eta_1 \sin\theta_1 \tag{2}$$

Expansion of 2 yields

$$(\eta_2 + \Delta\eta_2)\ (\sin\theta_2 + \Delta\theta_2\cos\theta_2) = \eta_1\sin\theta_1 \tag{3}$$

It has been assumed that $\Delta\theta_2$ is a very small change due to a very small change, $\Delta\eta_2$, in η_2. Then

$$\sin\Delta\theta_2 \simeq \Delta\theta_2$$

and

$$\cos\Delta\theta_2 \simeq 1$$

Thus, from 3 one obtains

$$\Delta\theta_2 = \frac{1}{\cos\theta_2}\left[\frac{\eta_1\sin\theta_1}{\eta_2 + \Delta\eta_2} - \sin\theta_2\right] \tag{4}$$

However, from Snell's law

$$\sin\theta_2 = \frac{\eta_1}{\eta_2}\sin\theta_1 \tag{5}$$

and

$$\cos\theta_2 = \left[1 - \frac{\eta_1^2 \sin^2\theta_1}{\eta_2^2}\right] \tag{6}$$

When Equations 5 and 6 are substituted into 4, one obtains

$$\Delta\theta_2 = \frac{\eta_1\,\eta_2 \sin\theta_1}{\left[\eta_2^2 - \eta_1^2 \sin^2\theta_1\right]^{1/2}} \left[\frac{1}{\eta_2 + \Delta\eta_2} - \frac{1}{\eta_1}\right] \tag{7}$$

but

$$\frac{1}{\eta_2 + \Delta\eta_2} = \frac{\eta_2 - \Delta\eta_2}{\eta_2^2 - \Delta\eta_2^2}$$

It should be noted that $\Delta\eta_2^2$ is extremely small compared to η_2^2 and can be neglected. Substitution of the above approximation into Equation 7 yields

$$\Delta\theta_2 = \frac{\eta_1 \sin\theta_1\,\Delta\eta_2}{\eta_2\left[\left(\eta_2^2 - \eta_1^2\sin^2\theta_1\right)^{1/2}\right]} \tag{8}$$

The light refracted will have a phase different from that of the incident light. For a medium having a definite thickness and for a fixed angle of incidence, the phase variation of the refracted light will be a function of the index of refraction of the medium.

Figure 2 represents a typical refractive situation. Regardless of whether

η_m is real or complex and for a wavelength λ, the medium of Figure 2 can be represented by the characteristic 2×2 matrix of the form

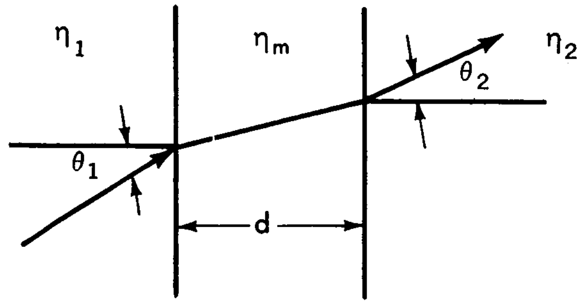


Figure 2–Optical Refraction.

$$\begin{bmatrix} \cos\theta & j\dfrac{\sin\theta}{\eta} \\ j\,\eta\sin\theta & \cos\theta \end{bmatrix} \qquad (9)$$

In this case,

$$\theta = k\,\eta_m\, d\cos\theta = \frac{2\pi}{\lambda}\,\eta_m\, d\cos\theta$$

The amplitude transmittance, T, can be derived using the elements of 9 to obtain

$$T = \frac{2\eta_1}{(\eta_1+\eta_2)\cos\theta + j\left(\dfrac{\eta_1\eta_2}{\eta_m} + \eta_m\right)\sin\theta} \qquad (10)$$

By multiplying numerator and denominator by the complex conjugate, T can be represented by

$$T = X - jY \qquad (11)$$

For this case,

$$X = \frac{2\eta_1(\eta_1+\eta_2)\cos\theta}{\left[(\eta_1+\eta_2)\cos\theta\right]^2 + \left[\left(\dfrac{\eta_1\eta_2}{\eta_m} + \eta_m\right)\sin\theta\right]^2}$$

$$Y = \frac{2\eta_1\left[\dfrac{\eta_1\eta_2}{\eta_m} + \eta_m\right]\sin\theta}{\left[(\eta_1+\eta_2)\cos\theta\right]^2 + \left[\left(\dfrac{\eta_1\eta_2}{\eta_m} + \eta_m\right)\sin\theta\right]^2}$$

Equation 11 can be written as

$$T = De^{j\phi} \tag{12}$$

where

$$D = \left[X^2 + Y^2\right]^{1/2} \tag{13}$$

and

$$\phi = \cos^{-1}\left[\frac{X}{\left[X^2 + Y^2\right]^{1/2}}\right] \tag{14}$$

Therefore, if the thickness of the medium is known and the wavelength of light is known, it appears that the phase is a function of the index of refraction of the medium. The above equation does not represent the complete story of transmittance of a coherent light beam through the atmosphere, however, the changing index of refraction of the atmosphere with altitude does represent one of the major problem areas in trying to position a narrow-beam LASER to a predetermined position in space.

II. RETARDATION EFFECTS

The effect of the changing index of refraction of the atmosphere is not taken into account when calculating the vector potential at a point due to a radiating source. The retarded vector potential is derived assuming a straight line distance from the source on the basis of integrating out time and substituting the point coordinates of the point at which the vector potential is to be calculated. This retarded vector potential is necessary to account for the fact that electromagnetic waves travel with a finite velocity. For a charge-free region, Maxwell's equations can be written in the following form:

$$\nabla \times \vec{B} = \mu \vec{J} + \mu \frac{\partial \vec{D}}{\partial t} \tag{15}$$

where

$$\vec{J} = \vec{J}' + \vec{J}''$$

and $\vec{J}'$ is due to the source and $\vec{J}'' = \sigma E$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \tag{16}$$

$$\nabla \cdot E = \frac{\ell}{\epsilon} \tag{17}$$

$$\nabla \cdot \vec{B} = 0 \tag{18}$$

Since the divergence of the curl of a vector is identically zero, Equation 18 can be derived from the curl of a vector A, or

$$\vec{B} = \nabla \times \vec{A} \tag{19}$$

A is the famous retarded vector potential of electromagnetic theory. Substitution of Equation 19 in Equation 16 yields

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0 \tag{20}$$

Since the curl of the gradient is identically zero

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = - \nabla \phi \tag{21}$$

Equation 21 gives rise to the Maxwell-Lorentz condition

$$\vec{E} = - \nabla \phi - \frac{\partial \vec{A}}{\partial E} \tag{22}$$

Thus, the electric field is a function of both a scalar and a vector potential. Expansion of Equation 15 to

$$\nabla \times \nabla \times \vec{A} = \nabla\nabla \cdot \vec{A} - \nabla^2 \vec{A}$$

and substituting Equation 22 yields

$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} - \mu\sigma \frac{\partial \vec{A}}{\partial t} = - \mu\vec{J}' + \nabla \left\{ \nabla \cdot A + \mu\epsilon \frac{\partial \phi}{\partial t} + \mu\sigma\phi \right\} \tag{23}$$

Equation 23 leads to the Lorentz condition

$$\nabla \cdot \vec{A} = - \mu\epsilon \frac{\partial \phi}{\partial t} - \mu\sigma\phi \tag{24}$$

For $\sigma = 0$

$$\nabla \cdot \vec{A} = - \mu\epsilon \frac{\partial \phi}{\partial t}$$

If Equations 22 and 24 are substituted into Equation 17, one obtains

$$\nabla^2 \phi - \mu\epsilon \frac{\partial^2 \phi}{\partial t^2} = - \frac{\ell}{\epsilon} \tag{25}$$

Equation 23 yields for the Lorentz condition and $\sigma = 0$

$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J}' \tag{26}$$

$\nabla^2 - \mu\epsilon\,(\partial^2/\partial t^2)$ is denoted as the four-dimensional D'Alembertian operator $\Box$

Equations 24, 25, and 26 can then be written as:

$$\nabla \cdot \vec{A} = -\mu\epsilon \frac{\partial \phi}{\partial t} \tag{27}$$

$$\Box\phi = -\frac{\ell}{\epsilon} \tag{28}$$

$$\Box\vec{A} = -\mu\vec{J}' \tag{29}$$

The D'Alembertian is essentially a four-dimensional Laplacian and indicates that for radiation one must take into account the dimension of time in addition to the three spacial dimensions.

If Equations 27 and 29 are multiplied by c, and Equation 28 by $\underline{i}$, then Equation 27 is the four-dimensional divergence operator operating on $(c\vec{A}, i\phi)$.

If we denote the vectors $\vec{\Phi}$ and $\vec{P}$ by

$$\vec{\Phi} = (c\vec{A},\ i\phi)$$

$$\vec{P} = \left(cu\vec{J}',\ i\frac{\ell}{\epsilon}\right)$$

then

$$\Box\Phi = -\vec{P} \tag{30}$$

$$\nabla \cdot \Phi = 0 \tag{31}$$

$$\Box\psi = 0 \tag{32}$$

where ψ is defined as

$$\vec{A}' = \vec{A} - \nabla\psi$$

$$\phi' = \phi + \frac{\partial\psi}{\partial t}$$

It is assumed that Φ decreases at least as fast as $1/R$ as R goes to infinity. In addition, Equation 32 is satisfied if

$$\psi \alpha \frac{1}{R^2}$$

When Green's Lemma is used to operate on ψ and Φ_j, the four-dimensional and three-dimensional integral expressions result.

$$\iiiint \nabla \cdot \left(\Phi_j \Box\psi - \psi\Box\Phi_j\right) d\tau_4 = \iiint \left(\Phi_j \Box\psi - \psi\Box\Phi_j\right) ds_3 \qquad (33)$$

but

$$\Box\psi = 0 \qquad (34)$$

$$\psi = \frac{1}{R^2} \qquad (35)$$

$$\Box\Phi_j = P_j \qquad (36)$$

Substitution of 34, 35, and 36 into 33 yields

$$\iiiint \frac{\vec{P}_j}{R^2} d\tau_4 = \iiint \left(\Phi_j \nabla \frac{1}{R^2} - \frac{1}{R^2} \nabla\Phi_j\right) ds_3 \qquad (37)$$

If Equation 37 is integrated around an infinite sphere and an infinitesimally small sphere at $R = 0$, the potential function at the center of the infinitesimal sphere is the only term that remains on the right half side of Equation 37. This yields

$$\Phi_j^{\circ} = \frac{1}{4\pi^2} \iiiint \frac{\vec{P}_j}{R^2}\, d\tau_4 \tag{38}$$

If the time variable is integrated out of 38, one obtains

$$\vec{\Phi}_j = \frac{1}{4\pi} \iiint \frac{[\vec{P}_j]}{R}\, ds \tag{39}$$

where $\vec{P}_j$ is taken at the time $t = t_0 - (R/c)$ and the observation point is a distance R from the source. If we let $\vec{\Phi}_j = \vec{A}$ and $\vec{P}_j = \mu J'$, Equation 39 expresses the retarded vector potential A.

$$\vec{A} = \frac{\mu}{4\pi} \iiint \frac{[\vec{J}]}{R}\, ds \tag{40}$$

The application of Equation 40 depends on the distribution of the current sheet J, but the retardation effect must be taken into account. The E and H fields can be obtained knowing A from Maxwell's equations. It should be noted that R is assumed as the straight line distance from an infinitesimal radiator having current density J.

Schelkunoff has introduced fictitious magnetic charges in his solution for the electric and magnetic fields. This is to account for the situations where the radiation is due to an electric field and/or a magnetic field. In his solution, E and H are broken up into two (2) components. The individual solutions are then added together through the Lorentz condition to give the two (2) vector potentials A and F; where

$$\vec{A} = \frac{\mu}{4\pi r} e^{j(\omega t - kr)} \iint \vec{J} e^{jkr'\cos\psi}\, ds \tag{41}$$

$$\vec{F} = \frac{\epsilon}{4\pi r} e^{j(\omega t - kr)} \iint \vec{M} e^{jkr'\cos\psi}\, ds \tag{42}$$

where for far field solutions

J = electric current density of source

M = magnetic current density of source

μ = permeability of medium

ϵ = permittivity of medium

ω = angular frequency of source

k = phase constant = $2\pi/\lambda$

r = distance from center of source coordinate system to the point where A and F are to be calculated

r' = distance from center of source coordinate system to infinitesimal radiating area

ψ = $\cos^{-1}\left(\vec{r}' \cdot \vec{r} / |r'| \, |r|\right)$ as shown in Figure 3

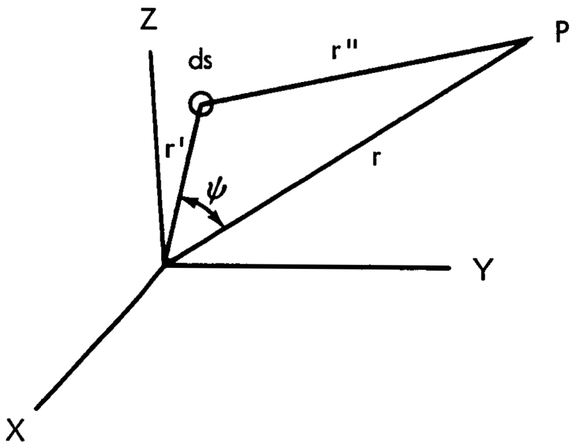


Figure 3–Geometry for Radiation Vectors.

The radiation vectors N and L are defined as

$$\vec{N} = \iint \vec{J} e^{jkr'\cos\psi} \, ds \qquad (43)$$

$$L = \iint \vec{M} e^{jkr'\cos\psi} \, ds \qquad (44)$$

It should be pointed out that Equations 43 and 44 are derived on the basis of the observation point staying fixed with respect to the source during the time r/c. On this basis, for far field considerations, it is assumed that the point is always far enough away from the source that the fields can be considered as plane waves.

A narrow beam LASER transmitting either from Earth to a deep-space vehicle or from a deep-space vehicle to Earth is an unusual situation due to the refractive properties of the atmosphere and the narrow beamwidths involved. Expressions 43 and 44 assume that ψ is fixed for a particular geometry. If the position of the observation point changes more than a beamwidth perpendicular to the beam during the time r/c, then $\cos\psi$ relative to the observation point is not fixed during the time r/c. The radiation vectors 43 and 44 can be considered independent of the motion of the spacecraft if the observation point at which the fields are desired is selected to coincide with the spacecraft position at the time $t_0 + (r/c)$ where t_0 is the time of radiation from the source and r is the range of the spacecraft at the time $t_0 + (r/c)$.

In order to appreciate the problem of propagation times to spacecraft on planetary missions, Table II gives the minimum and maximum propagation times to some of the closer planets. The minimum and maximum times are due to the orbital eccentricity of the various planets.

Table II

Propagation Times to Planets

Planet	Max	Min
Mercury	11.53 min	5.08 min
Venus	14.3 min	2.3 min
Mars	21.1 min	4.35 min
Jupiter	5.16 min	35 min
Saturn	1.46 min	1.18 hr.
Uranus	2.8 min	2.52 hr.

The times above have some uncertainty associated with them due to the error magnitudes in the velocity of light and position of the planets. Neptune and Pluto have been omitted due to the great distances involved.

There are very many problem areas to be resolved before a manned mission to even Mars will be feasible. However, it appears that these are not insurmountable, and in this respect, it is necessary to consider communications systems for supporting such a mission. All of the classical laws of electromagnetic wave theory apply to the transmission of coherent light at great ranges, however, the effects of atmospheric refraction will probably have to be measured experimentally before a narrow beam LASER can be considered for deep-space applications.

III. SPACIAL BEAM OFFSET CONSIDERATIONS

The propagation times and Fraunhoff diffraction pattern must be tied together to insure that the beam will intercept the spacecraft telescope in order to optimize communications at long ranges. In the case of a LASER transmitting system operating in conjunction with a large receiving telescope, one is usually concerned with 2 relatively narrow beams. The power density in the far field from a circular aperture can be expressed as a general equation for the Fraunhoff diffraction field as

$$S_r = K\left[\frac{J_1(kR\sin\theta)}{kR\sin\theta}\right]^2 \cos^4\frac{\theta}{2} \tag{45}$$

where K depends on geometry and the field configuration θ is the angle off of the beam axis

k = phase constant

R = radius of the aperture.

Equation 45 is plotted in Figure 4 for the main beam and first two (2) sidelobes. This plot is made in the focal plane of the aperture.

Figure 4 assumes very small angles so that $\cos^4(\theta/2) \simeq 1$. For this approximation

$$kR\sin\theta \simeq kR\theta \tag{46}$$

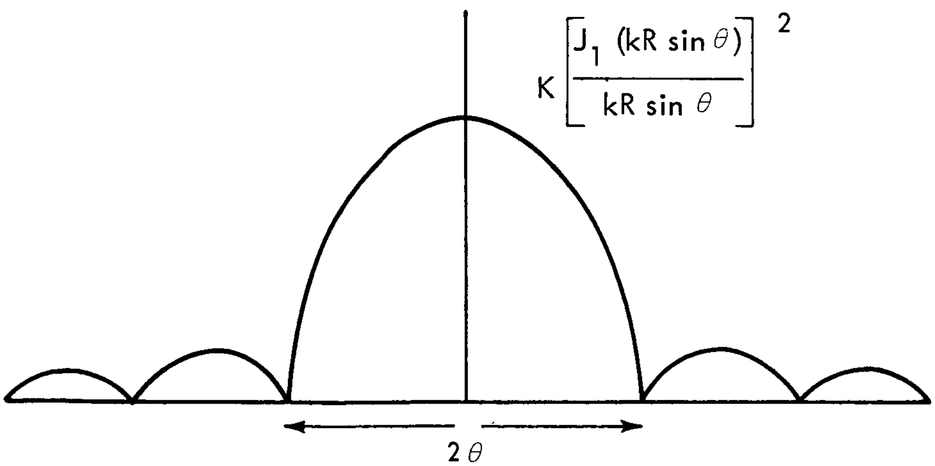


Figure 4–Plot of Fraunhoff Diffraction Pattern.

The first zero of J_1 $(kR \sin \theta)$ occurs when

$$kR\theta = 3.83 \tag{47}$$

or

$$\theta = 1.22 \frac{\lambda}{D}$$

where

$$k = \frac{2\pi}{\lambda}$$

and

$$D = 2R$$

Equation 47 is the approximate expression for the half angle of the primary beam in the Fraunhoff diffraction field for a circular source.

When considering present missions using microwave antenna systems, Equations 41 and 42 can be used straightforward for a particular geometry by replacing t in $\vec{J}$ and $\vec{M}$ by $t_0 - (r/c)$ where r is the range to the observation point (spacecraft) P at the time t_0. For these systems, it can be assumed for far field considerations that the change in ψ during the one-way propagation time is not significant to affect the signal power in the far field at the spacecraft.

Consider an 85-foot circular aperture emitting at a frequency of 2300 Mc. Using the approximation that $\theta = 1.22 (\lambda/D)$ results in approximately a 3 db beamwidth of 6×10^{-3} radians.

The diameter of the area subtended by this angle at 5×10^7 miles is 300,000 miles. A spacecraft moving at 5 miles/second perpendicular to the electrical axis of the aperture would move about 1400 miles during the propagation time of the wave from the source to the spacecraft. Thus, if the center of the Fraunhoff diffraction pattern is centered on the spacecraft at $t = t_0$; then at $t = t_0 + (r/c)$, the radiation pattern will still be maximized at the spacecraft for all practical purposes. Thus, for present systems, the retarded vector potential as defined is adequate to determine the electric and magnetic fields at the spacecraft for real time antenna positioning.

Future systems for deep manned space missions, for example, the first manned mission to Mars, might utilize narrow beam LASERs for ground-to-spacecraft and/or spacecraft-to-ground communications. Later missions might utilize highly maneuverable spacecraft with a very accurate guidance system in order to eliminate the tracking requirement. However, large information bandwidths would seem a necessity from spacecraft-to-ground, even in later missions. For this reason, it is interesting to consider one aspect of the problem of positioning the beam. Assume a LASER with a 0.1 sec beamwidth operating at 5×10^{14} cps. For the sake of isolating one problem, assume negligible atmospheric refraction and turbulence. The diameter of the area subtended by this angle at 5×10^{7} miles is approximately 25 miles. A space vehicle moving at 5 miles/second perpendicular to the beam would travel approximately 1,350 miles during the one-way propagation time of an electromagnetic wave. Thus, if the vehicle were moving perpendicular to the LASER beam axis, the LASER would have to be pointed ahead of the vehicle trajectory approximately 54 beamwidths. Future communications systems utilizing optical LASERs must develop accurate methods for predicting spacecraft position in order to insure that the main lobe of the far field pattern intercepts the spacecraft telescope.

Figure 5 indicates the relative position of a LASER and a spacecraft for two (2) different times t_1 and t_2. At the time t_1, the spacecraft is at position 1 and at

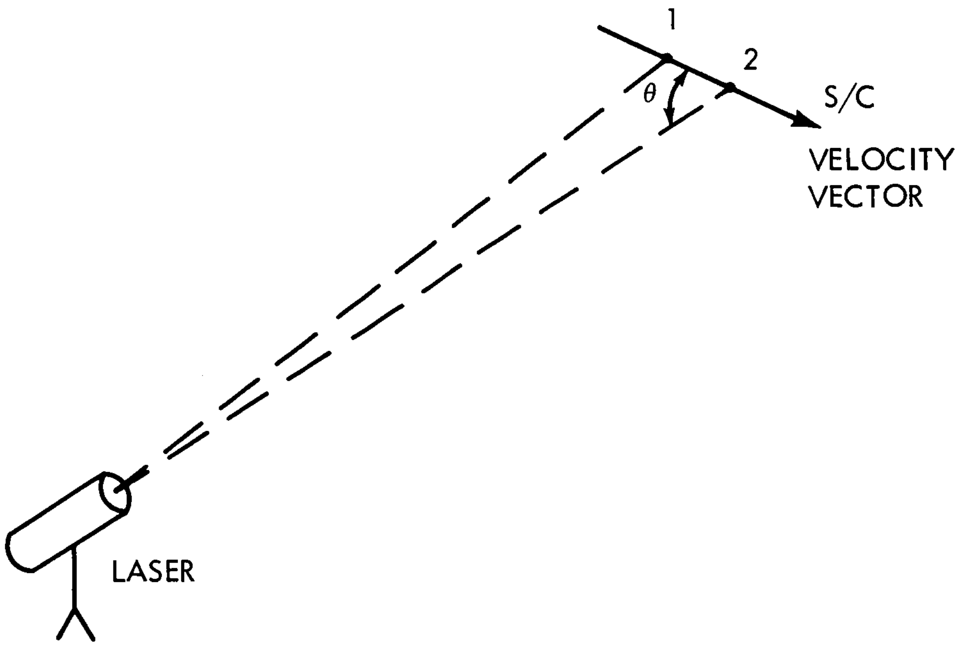


Figure 5–Spacecraft - LASER Geometry.

the same time the LASER transmits a short pulse. Assume the coordinates of the spacecraft with respect to the LASER at t_1 are

$$S_1 = (0, 0, Z_1) \tag{48}$$

It is assumed that the spacecraft at position 1 is directly on line with the electrical axis of the telescope. A new set of reference axes can now be placed at $Z = Z_1 = R$ at the time t_1 where the coordinates of the spacecraft at t_1 in reference axis 2 are

$$S_2 = (0, 0, 0) \tag{49}$$

If the velocity components of the spacecraft are dX/dt, dY/dt, and dZ/dt, then at a time t_2 the spacecraft will be at the position coordinates relative to the frame of reference 2

$$S_2 = \left(\frac{dX}{dt}\Delta t, \frac{dY}{dt}\Delta t, \frac{dZ}{dt}\Delta t\right) \tag{50}$$

where

$$\Delta t = t_2 - t_1$$

In this case, t_2 is the one-way propagation time from the LASER to the spacecraft. Originally, the spacecraft was at a slant range of $R_1 (Z_1)$ relative to the LASER at $t_1 = 0$. At the time t_2, the spacecraft is at the new slant range of $R_1 + (dZ/dt) t_2$ relative to the LASER. It is imperative that for reliable communications, the intersection of the beam and spacecraft occur at the same time t_2. For this case

$$t_2 = \frac{R_2}{c} \tag{51}$$

where

$$R_2 = R_1 + \frac{dZ}{dt}\Delta t$$

Thus, the beam of a very narrow LASER must be positioned at the time (t_0) the source radiates to a point in space determined by not only the coordinates of the spacecraft at the time t_0, but also the velocity components of the spacecraft to insure that at the time t_2 the maximum of the main lobe of the far field pattern intersects the spacecraft. Consequently, it will be necessary that predicted information on spacecraft range, velocity and angle be extremely accurate for a reliable communications system.

The only case where this would not be a problem is when $dX/dt = dY/dt = 0$ In this case, the spacecraft is flying a trajectory which is on line with the axis of the LASER beam and a spacial offset is not required. Due to the trajectory that a spacecraft must fly on a planetary mission and due to the ground station location and changing aspect angle, this geometry would not occur for the entire duration of a mission. The worst case situation occurs when the spacecraft is flying nearly perpendicular to the axis of the telescope. In this case, $dZ/dt \quad 0$, and the slant range R_2 is approximately equal to R_1 . Thus, one becomes concerned with the values of dX/dt and dY/dt .

For the purpose of determining spacial offset, either dX/dt or dY/dt can be taken as zero. If dZ/dt and $dY/dt = 0$, then the spacecraft is flying essentially parallel to the X axis. If α , β, and P are the angles between the spacecraft velocity vector and the X, Y, and Z axes of reference frame 2 respectively, then at time t_1 ;

$$\alpha_1 = \cos^{-1} \frac{\frac{dY}{dt}}{V} \tag{52}$$

$$\beta_1 = \cos^{-1} \frac{\frac{dX}{dt}}{V} \tag{53}$$

$$\rho_1 = \cos^{-1} \frac{\frac{dZ}{dt}}{V} \tag{54}$$

where

$$V = \sqrt{\left(\frac{dX}{dt}\right)^2 + \left(\frac{dY}{dt}\right)^2 + \left(\frac{dZ}{dt}\right)^2} \tag{55}$$

In order to simplify the problem without destroying the significance, it is assumed that $d\alpha/dt = d\beta/dt = d\ell/dt = 0$ during the one-way propagation time of the rf wave. If t_1 is the time when a wave front emanates from the LASER, then at t_2

$$\alpha_2 = \alpha_1, \qquad \beta_2 = \beta_1, \qquad \rho_2 = \rho_1 \tag{56}$$

The steering of a LASER beam to a deep-space vehicle must take into account a factor that has not been a serious problem with standard microwave antenna systems. This is the beam spacial offset required as a function of the dynamics of the spacecraft. Due to the small angles θ involved, the average diameter of the area subtended by the main lobe of the Fraunhoff diffraction field can be approximated by

$$D_{ave.} = \left(\frac{R_1 + R_2}{2}\right)\theta \tag{57}$$

where R_1 and R_2 are the ranges at the time the source radiates and the range after the one-way propagation time to the spacecraft. The only case where $R_1 = R_2$ is when $dZ/dt = 0$. Equation 57 represents an average diameter of the main beam.

Figure 6 represents the geometry of a spacecraft relative to the LASER at the ranges R_1 and R_2.

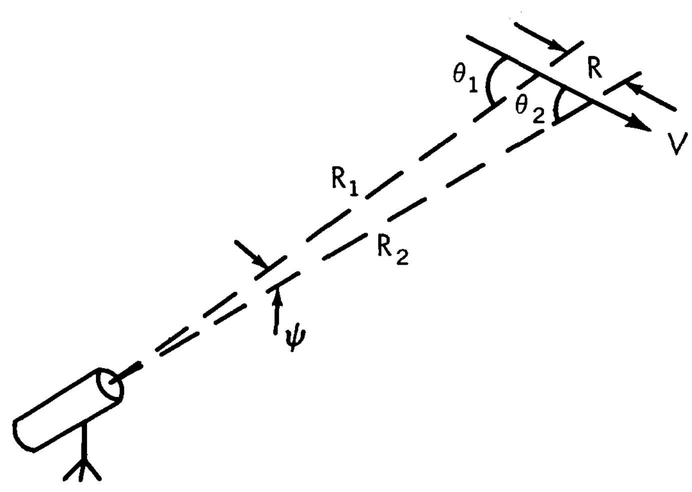


Figure 6–Relative Motion of Spacecraft with Respect to the LASER at Ranges R_1 and R_2.

The radial velocity component at the range R_1 is equal to $V \cos \theta_1$, while the tangential velocity component is $V \sin \theta_1$. At R_2, these are respectively, $V \cos \theta_2$ and $V \sin \theta_2$ where it is assumed that $V_1 = V_2 = V$. The angle ψ as shown in Figure 6 is equal to

$$\psi = \pi - [\pi - \theta_1] - \theta_2 = \theta_1 - \theta_2 \tag{58}$$

The distance R in Figure 6 can be obtained from the law of cosines as

$$R = \sqrt{R_1^2 + R_2^2 - 2R_1 R_2 \cos \psi} \tag{59}$$

For deep space considerations

$$R_1 \simeq R_2$$

then,

$$R \simeq R_1 \sqrt{2 - 2 \cos \psi} \tag{60}$$

However, R is also approximated by,

$$R = V\left(\frac{t_2 + t_1}{2}\right) \tag{61}$$

where $t_2 + t_1/2$ is the average propagation time and $t_1 = (R_1/c)$; $t_2 = (R_2/c)$. Equations 60 and 61 yield

$$t_2 = \left(\frac{R_1}{c}\right) - \left(\frac{2R_1}{v}\right) \sqrt{2 - 2 \cos \psi} \tag{62}$$

It can be seen from Equation 62 that for very small angles,

$$t_2 \simeq \frac{R_1}{c} \tag{63}$$

The spacecraft range at t_2 is

$$R_2 = R_1 + v \cos \theta_1 \left[\frac{t_2 + t_1}{2}\right] \tag{64}$$

During the time $t_2 + t_1/2$, the spacecraft's slant range will change by approximately

$$\Delta R_r = v \left[\frac{t_2 + t_1}{2}\right] \cos \theta_1 \tag{65}$$

In addition, during the same time the spacecraft will move perpendicular to the beam axis an amount equal to

$$\Delta R_p = v \left[\frac{t_2 + t_1}{2}\right] \sin \theta_1 \tag{66}$$

The diameter of the areas subtended by a narrow-beam LASER was approximated in Equation 57 as

$$D = \left[\frac{R_1 + R_2}{2}\right] \theta$$

where $R_1 + R_2/2$ was the average range to the target during the one-way propagation time and θ is the half angle of the main beam, but it is also used as an approximation of the 3 db beamwidth. In order to determine the spacial offset required of the LASER beam, one must take the ratio of Equations 66 to 57. Spacial offset in beamwidths is

$$B = \frac{\Delta R_P}{D} = \frac{v \left[\frac{t_1 + t_2}{2}\right] \sin \theta_1}{\frac{R_1 + R_2}{2} \theta} \tag{67}$$

It should be noticed that $R_1 + R_2/2$ can be written as $ct_1 + ct_2/2$. Substitution in Equation 67 yields

$$B = \frac{v \sin \theta_1}{c\theta} \tag{68}$$

Equation 68 is a very simple expression, independent of range. The spacial offset is essentially a function of the spacecraft velocity, aspect angle, and the width of the main beam in the far-field pattern.

Equation 68 is plotted as a function of aspect angle in Graphs 1 through 7, for a spacecraft velocity of 10,000 M/sec. It can be seen from Graph 1 that conventional microwave systems effectively communicate with a spacecraft in real time position. On the other hand, Graphs 2, 3, and 4 indicate that a LASER has to be offset considerably in beamwidth particularly for aspect angles approaching 90°

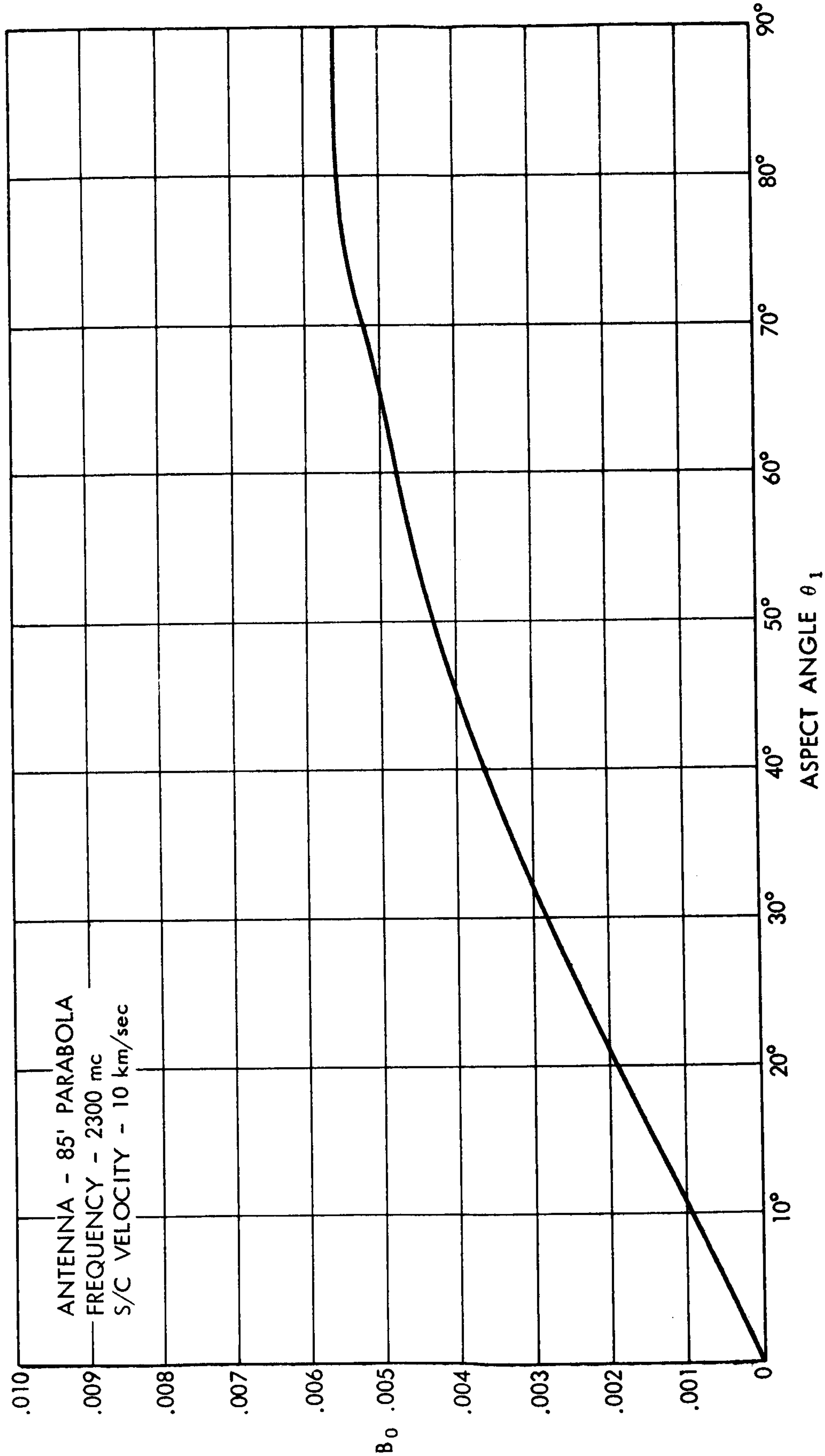
The complete problem from the retardation point of view involves both the transmitting LASER and the receiver telescope. The LASER on the ground or on a relay platform must be offset according to Equation 68. In addition, a diffraction-limited telescope on the spacecraft or ground must be able to position the telescope very accurately in order to intercept the maximum of the far-field radiation pattern of the LASER transmitter with the maximum of the diffraction pattern of the telescope.

A LASER having a beamwidth of 10 seconds of arc requires the least offset of those considered. On the other hand, a 0.1 second LASER beamwidth would require as much as 67 beamwidths of spacial offset for 90° aspect angles. Graphs 5, 6, and 7 are plots of the spacial offset required for a receiving telescope. For example, an earth-based telescope receiving from a LASER on a spacecraft has a similar problem to the LASER transmitter. Large telescopes have very narrow beamwidths and in order to maximize the received energy, it is necessary that the center of the telescope diffraction pattern be positioned to coincide with the maximum of the LASER diffraction pattern. Graph 7 indicates that a 100 cm telescope is comparable to a LASER having a beamwidth of 1 second of arc as far as spacial offset is concerned.

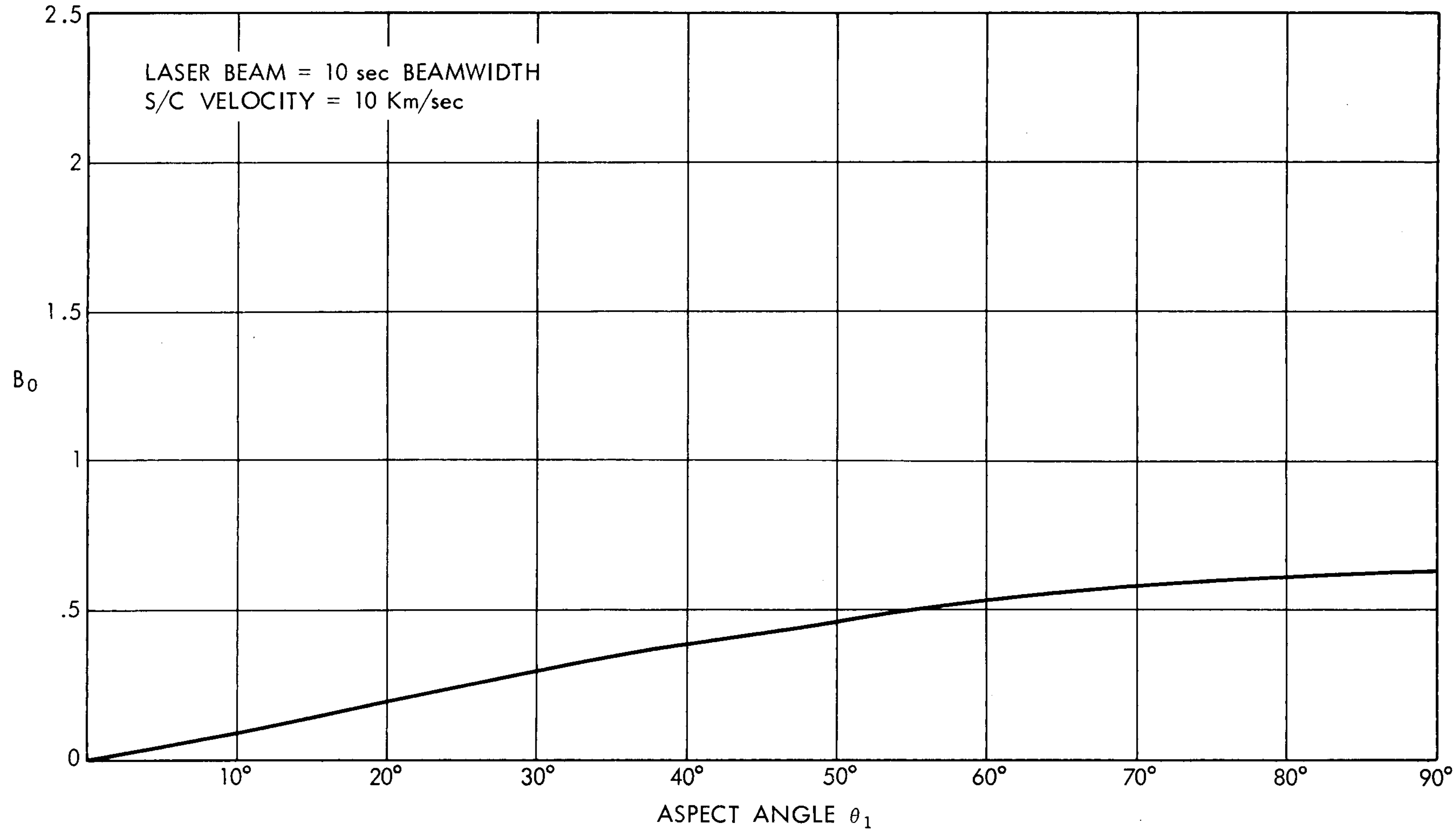
IV. SUMMARY

A LASER has certain advantages over microwave systems for communications at planetary distances. These aspects were not covered in this paper, but an attempt has been made to present one of the problem areas involved in steering either an earth-based LASER to a deep-space vehicle or steering a LASER on a relay station to a deep-space vehicle. It has been shown that for very narrow-beam LASERs it is necessary to apply a spacial beam offset to the transmitter in order that the energy in the main lobe of the diffraction pattern intersects the spacecraft. The amount of offset required is essentially independent of range and depends on spacecraft velocity, aspect angle, and the beamwidth of the LASER.

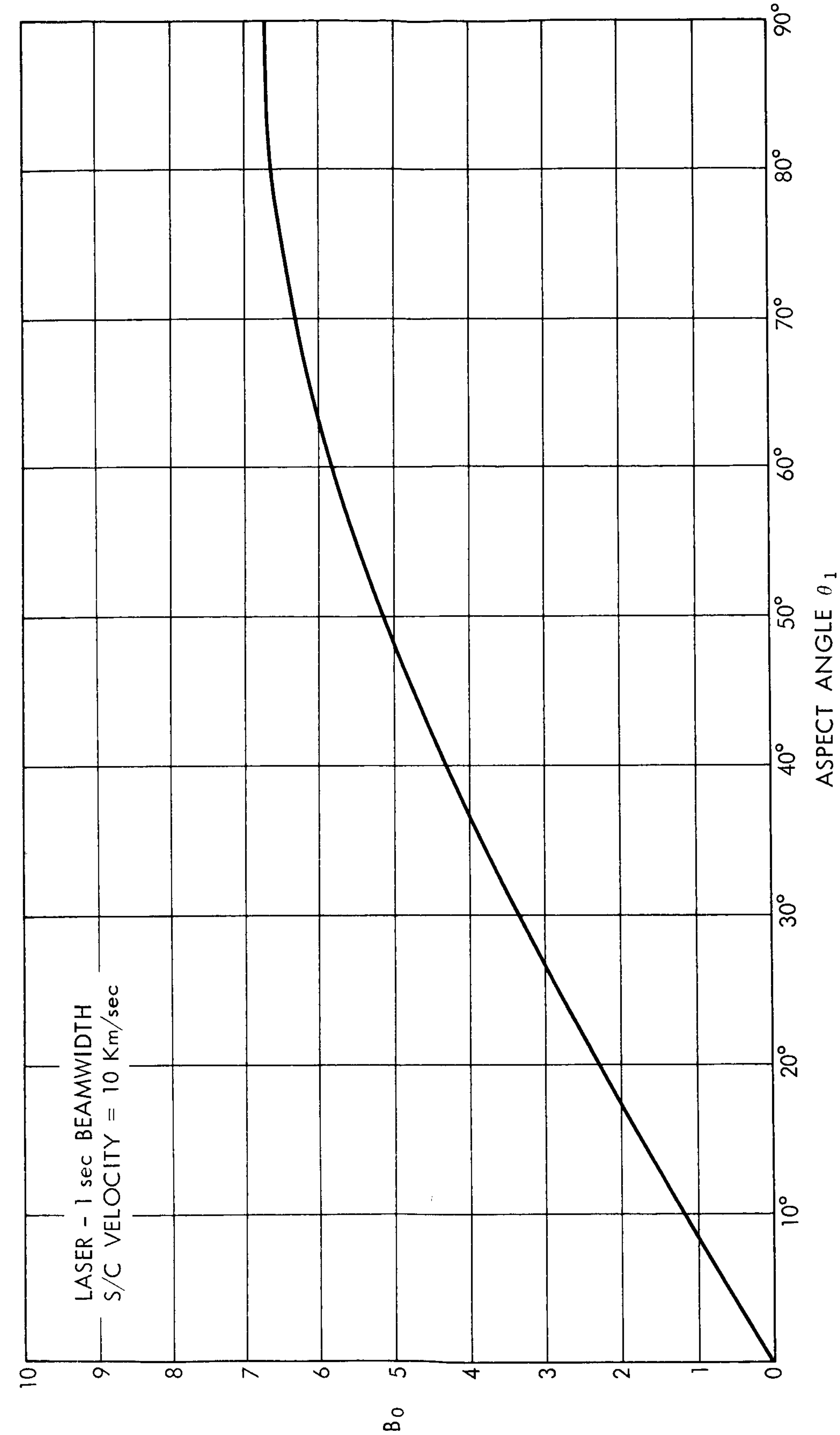
In order to properly position the LASER beam, velocity and angle information will have to be exceptionally accurate and extremely accurate servo systems will be required. It well may be that a completely new system will have to be invented in order to steer a narrow-beam LASER. The state-of-the-art is advancing at such a rate in LASER technology that the problems that seem insurmountable today will most likely be solved in the near future.


ANTENNA - 85' PARABOLA
FREQUENCY - 2300 mc
S/C VELOCITY - 10 km/sec
B_0
.010
.009
.008
.007
.006
.005
.004
.003
.002
.001
0
10°
20°
30°
40°
50°
60°
70°
80°
90°
ASPECT ANGLE θ_1

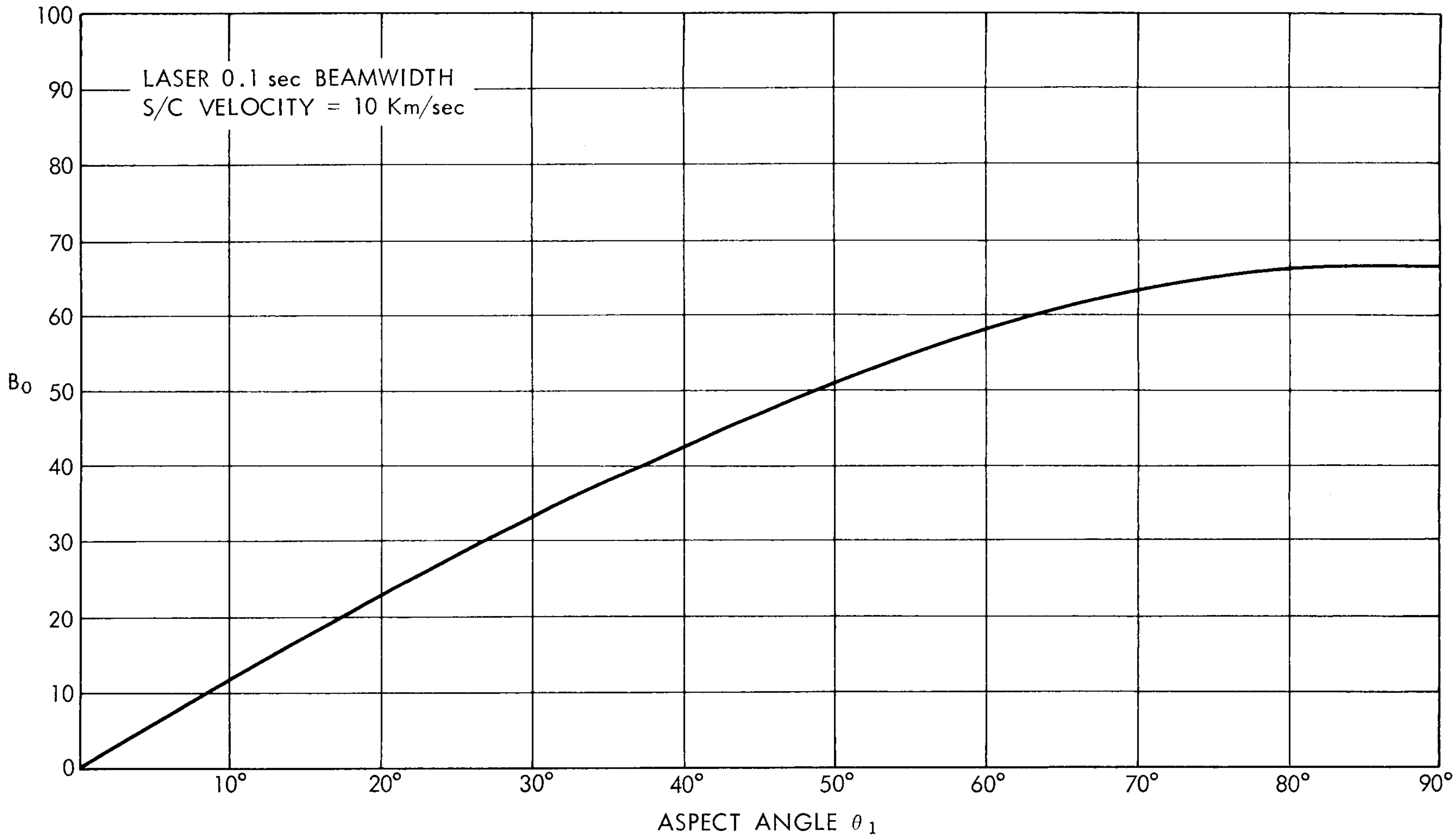
Graph 1.


LASER BEAM = 10 sec BEAMWIDTH
S/C VELOCITY = 10 Km/sec
2.5
2
1.5
1
.5
0
B_0
10°
20°
30°
40°
50°
60°
70°
80°
90°
ASPECT ANGLE θ_1

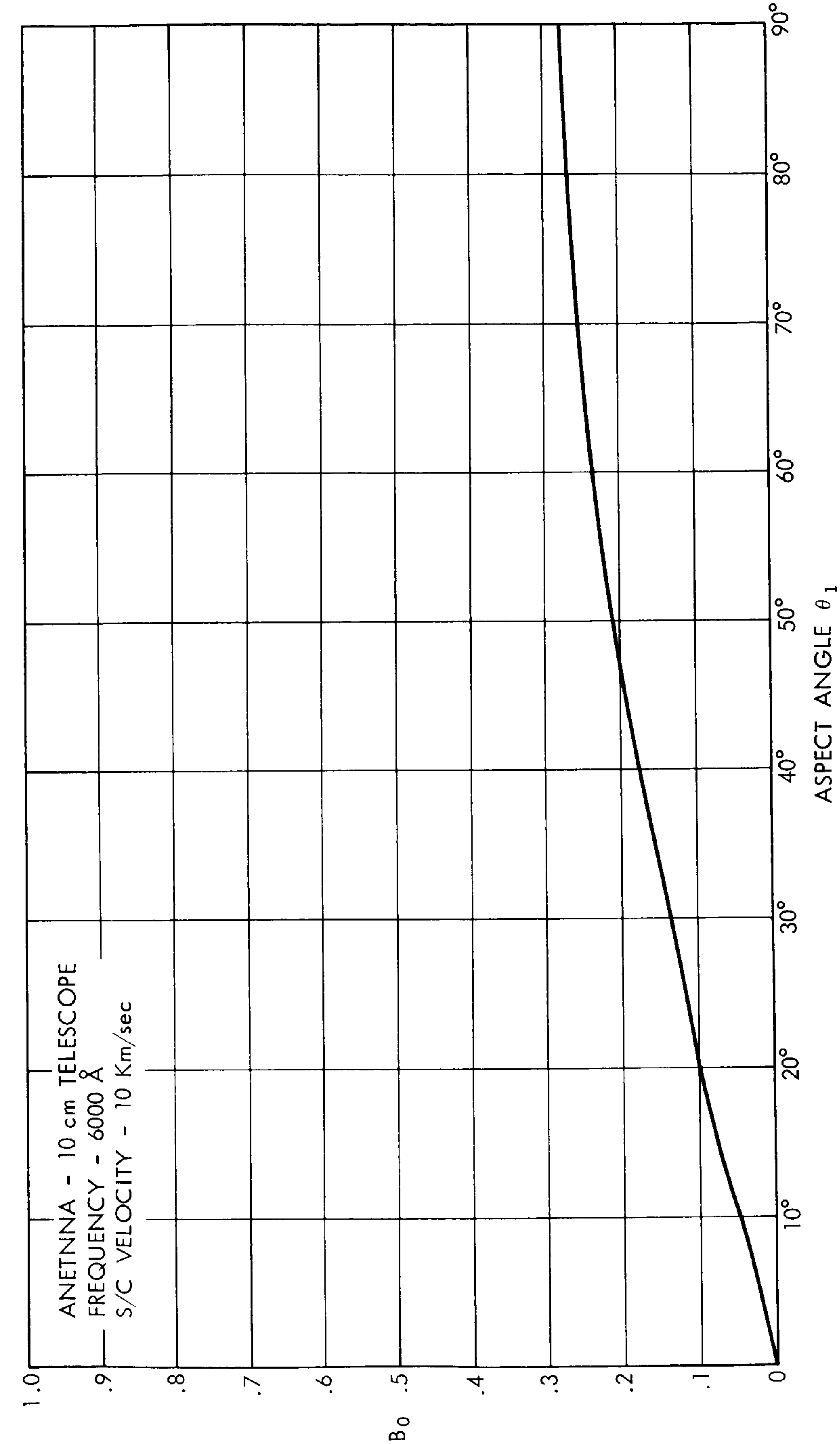
Graph 2.


LASER – 1 sec BEAMWIDTH
S/C VELOCITY = 10 Km/sec
B_o
0
1
2
3
4
5
6
7
8
9
10
10°
20°
30°
40°
50°
60°
70°
80°
90°
ASPECT ANGLE θ_1

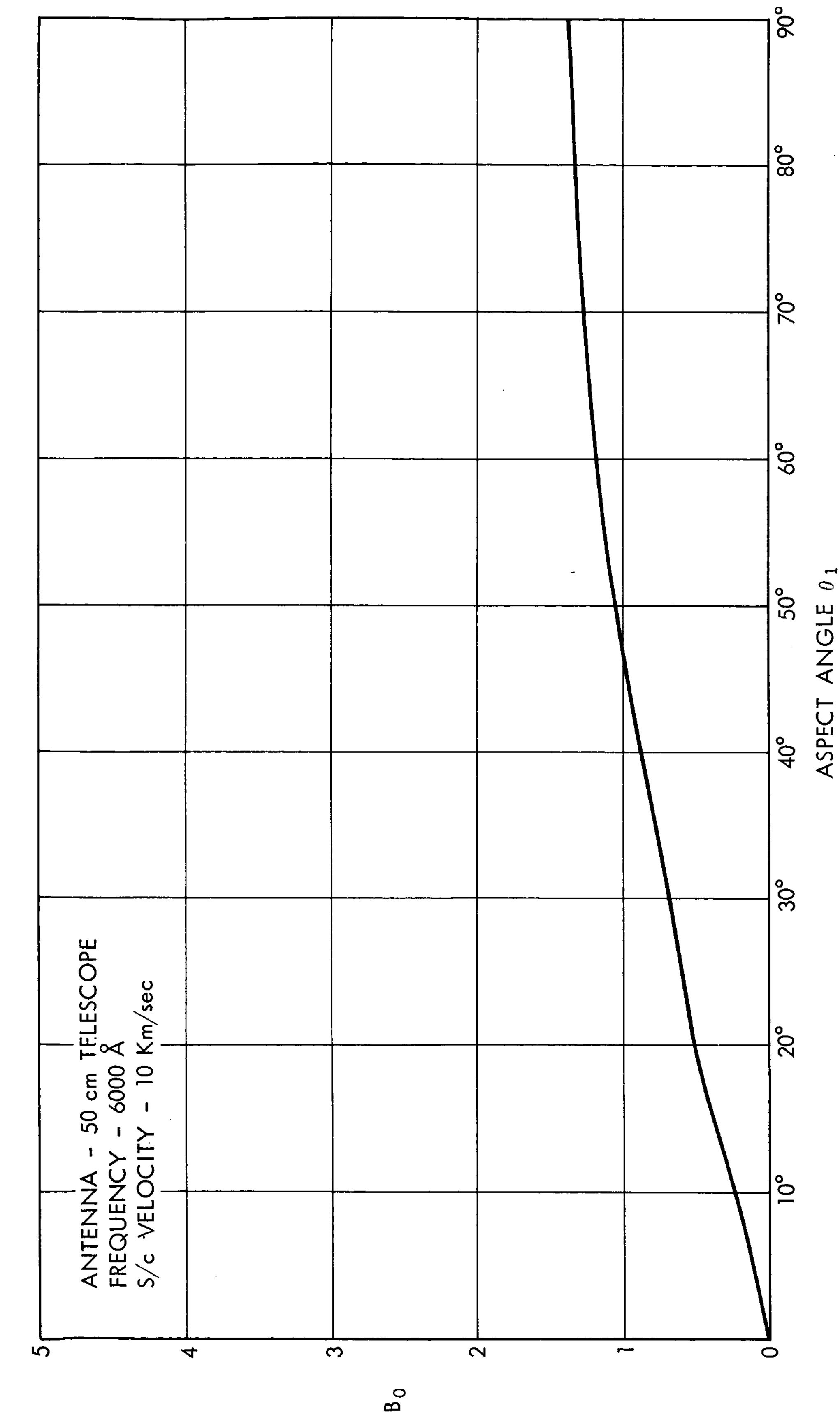
Graph 3.


LASER 0.1 sec BEAMWIDTH
S/C VELOCITY = 10 Km/sec
100
90
80
70
60
50
40
30
20
10
0
B_0
10°
20°
30°
40°
50°
60°
70°
80°
90°
ASPECT ANGLE θ_1

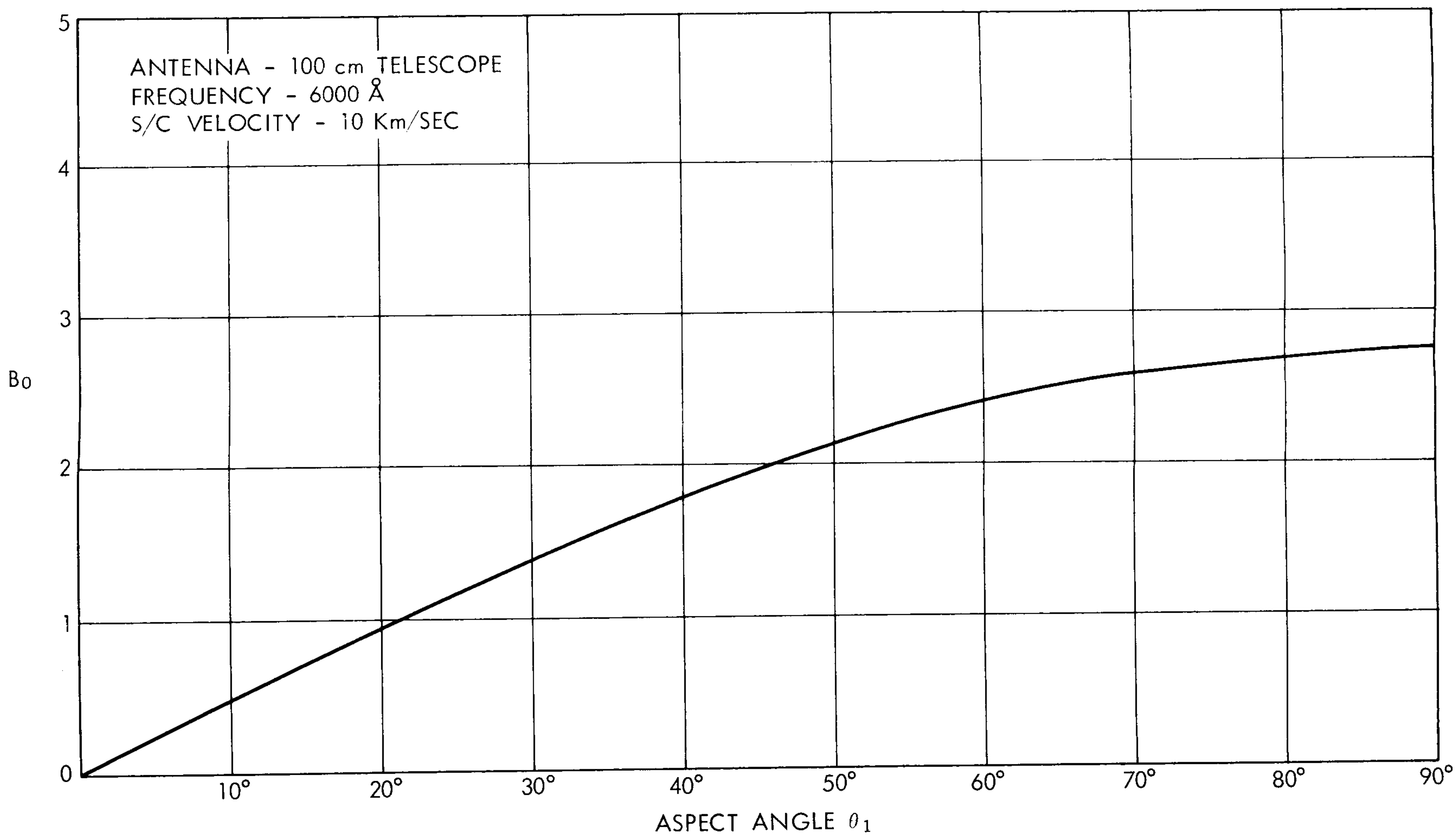
Graph 4.


ANETNNA - 10 cm TELESCOPE
FREQUENCY - 6000 Å
S/C VELOCITY - 10 Km/sec
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
B_o
10°
20°
30°
40°
50°
60°
70°
80°
90°
ASPECT ANGLE θ_1

Graph 5.


ANTENNA - 50 cm TELESCOPE
FREQUENCY - 6000 Å
S/c VELOCITY - 10 Km/sec
B_0
0
1
2
3
4
5
10°
20°
30°
40°
50°
60°
70°
80°
90°
ASPECT ANGLE θ_1

Graph 6.


ANTENNA - 100 cm TELESCOPE
FREQUENCY - 6000 Å
S/C VELOCITY - 10 Km/SEC
Bo
0
1
2
3
4
5
10°
20°
30°
40°
50°
60°
70°
80°
90°
ASPECT ANGLE θ_1

Graph 7.